CT Lab - MedPhys 501

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1 Introduction

Over the course of the semester, we have observed the general nature and theoretical concepts of radiation and medical physics through lectures and literature. This lab's purpose is to provide us with a more physical visualization of the concepts we've learned over the past four months by observing the physics behind a GE Discovery CT750 HD CT scanner.In particular this lab aims to reveal the effects of radiation from X-Rays and how a patient receives dose from the imaging process.

2 Lab Process

The lab procedure then involved testing a 6-cm^3 general purpose ion chamber to measure the exposure of a single-rotation axial CT scan. A brief introduction to the CT machine was given, and was then followed by an explanation of the CT scan parameters for the single-rotational scan. The machine operated in an axial mode with an applied voltage of 120 kV and an applied current of 100 mA, had a rotation time of 0.5s, and it's frame of view was of the head. In addition, the beam collimation was approximately 40mm, and had a slice thickness of 2.5mm. The ion chamber was then placed in the path of the x-ray beam as shown in figure 1(b) in order to gather the exposure values. During this process the electrometer was set to the "auto-dose" method, which allowed for a quick reset of the electrometer after the reading was recorded. After this process was conducted five times, the ion chamber was then removed from the path of the x-ray beam and the scan was again conducted five times, with five corresponding values. The data from both cases was then analyzed to determine the mean and variance for each situation, which can be found in Table 1.

2.1 Question 1

The procedure for both cases was conducted five times to reduce the uncertainty in the results of the measurement. While we expect to receive an exact value from a deterministic process, the issue with experimental science is that data can often be quite difficult to reproduce from a single measurement due to potential unexpected outside factors. Therefore, it is necessary, and often helpful for the scientist as well, to conduct many trials of the same experiment to reduce the aforementioned sources of error.

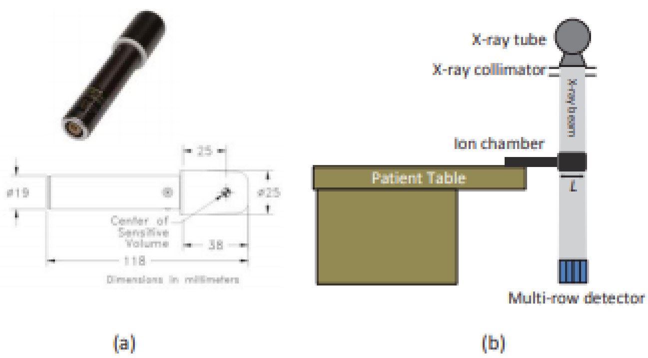

Figure 1: (a) The schematic of the 6-cm^3 general purpose ion chamber. (b) The in-air exposure measurement with the ion chamber inside of the collimated scanning region. (c) The in-air exposure measure with the ion chamber outside of the collimated scanning region.

Table 1: Table of the exposure for the two different setups with the 6-cm^{3}ionchamber.

Chamber Position	Test 1	Test 2	Test 3	Test 4	Test 5	Mean	Variance
Within Collimation	2.567 R	2.578 R	2.578 R	2.572 R	2.575 R	2.574 R	$2.15 \times 10^{-5} R^2$
Outside Collimation	52.08 mR	52.54 mR	52.56 mR	52.54 mR	52.64 mR	52.47 mR	0.05 mR^2

It is clear from the table that even without being directly underneath the x-ray beam collimation, the ion chamber still was exposed to non-negligible amounts of radiation. Therefore the question arises: "what would be possible mechanisms that could lead to ionizing radiation dose depositions outside of the beam"?

2.2 Question 2

There are three primary sources of possible background radiation that could be picked up by the ion chamber outside of the collimated beam:

1. Scattering from the air and bore on the the chamber:

   There would be some scattering of the X-Rays with the air and the bore of the chamber as the beam travels from the source to the ion chamber. Often, these scatterings are neglected in the math due to how small in magnitude they actually are.

2. Penumbra:

   In the case of the schematic of figure 1, the x-ray beam is treated as a point source, which continues until it reaches the target. However in real life this would not happen because the beam would spread as the source is 1 mm thick. This therefore implies that excess x-rays could be deposited into areas around the target location, which presents the most significant source of excess exposure and dose.

3. ”Bouncing” electrons - Off Focus Radiation:

   When x-rays are being generated at the source in the form photons at an anode, some photons will not completely stop upon reaching the target and instead will follow the focal path. This gives the idea of a “bouncing” photon as the photon can recoil off the target and curve further outward towards the cathode surrounding the target. This occurs due to the electromagnetic field between the the anode and cathode, and eventually the photons will once again curve towards the anode but in the new off-target location. This is a secondary form of exposure that is not as significant as the penumbra beam, but still must be considered.

The process of using a single general-purpose ion chamber is that with each CT image, the operator must image one section at a time. The process of observing a single cross-sectional area and then moving to the following slice is often quite time-consuming and can lead to a larger dose to the patient given that the tails of the exposure spectrum often overlap, increasing the dose to the patient.

Instead, often times a multi-rotational CT scan with a pencil chamber is used in order to minimize the ”leakage” that would occur from the “exposure tails”, therein minimize the excess dose to the patient. The pencil chamber is a 100mm long device that allows the investigator to directly compute the mean exposure level along the 100mm it covers. The mean exposure for the pencil chamber can be derived in the equations below:

$$X_{pencil} = \frac{\int_{-50mm}^{50mm} X(z)dz}{\int_{-50mm}^{50mm} dz}$$
$$= \frac{1}{100}\int_{-50mm}^{50mm} X(z)dz \tag{1}$$

This equation also allows for a direct calculation of the total exposure from a radiation source, which is to say that

$$X_v^{total} = \frac{100}{L} X_{pencil} \tag{2}$$

where L is the beam collimation width and has units of millimeters.

2.3 Question 3

In the case where sequential axial scans are taken, equation (2) would overestimate the total radiation exposure within the collimated x-ray beam. This is the case because of the previously mentioned tails on the exposure spectrum. With each sequential axial scan over a small distance “L”, there are two regions on the spectrum that reach into adjacent portions of length ”L”. Through repeated axial scans, these excess exposure readings will build up and provide an overestimate of the radiation exposure in each portion.

The pencil beam was then used in order to determine a localized expression of the exposure without any of the excess leakage terms.

Table 2: Table of the exposure for the trial with the collimated x-ray beam and the pencil chamber.

Trial	1	2	3	4	5	Mean	Variance
Exposure	1.111 R	1.108 R	1.115 R	1.115 R	1.110 R	1.112 R	9.7 $\times 10^{-6} R^2$

This table shows the fact that there is nearly a 50% decrease in overall exposure just from using a pencil chamber compared to the multi-purpose chamber.

2.4 Question 4

Before conducting dose-to-air calculations, the justification of charged particle equilibrium for the pencil chamber needs to be established, as without it, it cannot be said that the collisional kerma in air is the same as the dose in air. In order for there to be charged particle equilibrium, the number of charged particles entering some given volume must be the same as the number of charged particles exiting the volume. This can occur if the volume is significantly smaller than the range of the charged particles, or if there happen to be internal ionizing radiation events that exit the volume. In this case, since the irradiated medium is air bore that has a radius of 35cm, with a small ion chamber cavity with a radius of 0.45cm, it can easily be seen that the the walls of the air bore and the cavity are negligible. This therefore allows for the assumption of charged particle equilibrium.

Under charged particle equilibrium (henceforth referred to as CPE), the claim can be made that

$$D_{air} = (K_c)_{air}$$

Which allows for a direct dose calculation. From previous reference material, it is known that the dose to air under CPE is just the mean ionization energy of air $(\frac{\bar{W}}{e})_{air}$ times the exposure in the air. Using equation (2), the general form of the Dose to air can be determined

$$D_{air} = \frac{100}{L} X_{pencil} \left(\frac{\bar{W}}{e} \right)_{air} \tag{3}$$

In this case the distance L is approximately 40 cm. The dose to the tissue can be found using the using Burlin cavity theory as the cavity's size is significantly larger than the range of the electrons travelling through it. For the polyenergetic spectrum of electrons ranging from 0 keV to 120 keV, the largest range, according to interpolating values from eSTAR on NIST, would be 2.214 $\times 10^{-2}$ g/cm^2. When compared to the the value of the thickness of the ion chamber times the density of air, or 1.103 g/cm^2, it is clear that the cavity is quite large compared to the range. From Burlin Cavity theory we can say that, if the air in v were replaced with human tissue, then the dose to that tissue could be found in the equation (4) below:

$$\frac{D_{tissue}}{D_{air}} = \frac{(K_c)_{tissue}}{(K_c)_{air}} = \frac{\left(\frac{\mu_{ab}}{\rho}\right)_{tissue}}{\left(\frac{\mu_{ab}}{\rho}\right)_{air}} \tag{4}$$

From equation (4), the dose to the tissue can be found, but first the dose to air has to also be found. To do this, it's easy to know that the mean ionization energy for the air is approximately 33.97 J/C. Additionally, the NIST database can be consulted for tissue to the find the mean mass absorption coefficient using a mean energy of 70 keV such that $\left(\frac{\mu_{ab}}{\rho}\right)_{air}$ is $2.724 \times 10^{-2} g/cm^2$ and $\left(\frac{\mu_{ab}}{\rho}\right)_{tissue}$ is $2.941 \times 10^{-2} g/cm^2$. Thus, table 2 can be consult to construct the table with the unknown mean doses to the air and tissue respectively.

Table 3: The table of calculated doses to air and tissue from the exposure determined in table 2 and equations (3) and (4)

Case	Mean	Variance
Exposure	1.112 R	9.7 $\times 10^{-6} R^2$
Dose to Air	0.0244 Gy	5.43 $\times 10^{-11} Gy^2$
Dose to Tissue	0.0263 Gy	586 $\times 10^{-11} Gy^2$

After this process conducted, the lab then proceeded with work on a phantom to simulate the process of taking a CT scan of a segment of a patient's body. To do so, the pencil chamber was placed inside various bores in a phantom near the center and edge in order to get the dose values at each location. The CTDI Phantom is used in this case as it helps with modelling parts of the human body such as the head or a limb and the phantoms usually have diameters of approximately 16cm or 32cm for each corresponding part.

2.5 Question 5

It is also important to know the maximal kinetic energy that an electron can obtain through the Compton effect. The equation for this process was given in lab to be equation (5)

$$E_e^{max} = h\nu\left(\frac{2\varepsilon}{1+2\varepsilon}\right), \quad \varepsilon = \frac{h\nu}{511keV} \tag{5}$$

Since the photon energy being used in this case is 70 keV, substituting it into equation (5) reveals the max energy transfer is as follows:

$$E_e^{max} = 70keV\left(\frac{2\left(\frac{70keV}{511keV}\right)}{1+2\left(\frac{70keV}{511keV}\right)}\right) = 15.05keV$$

This is important to know as the it allows for a clear value to determine the R_{CSDA} of the photoelectrons in this case. This process also allows for a clearer confirmation that the cavity is effectively large due to dominance of the low energy electrons that occur from the Compton effect. This process is done to simulate the conditions of the human body as best as possible.

To calculate the dose to air and the dose to pmma a similar process as before can be used suchthat the dose to air can be found from the charge of the pencil chamber over the mass of the pencil chamber times 100cm over the length of the testing portion (again 40cm) all times $\left(\frac{\bar{W}}{e}\right)_{air}$. This is shown in equation (6) below:

$$D_{air} = \frac{100cm}{L}\frac{Q_{pencil}}{m_{pencil}}\left(\frac{\bar{W}}{e}\right)_{air} \tag{6}$$

Equation (6) can also be used with Burlin Cavity theory, this time using the mass energy-absorption coefficients for air and pmma at 70 keV such that

$$D_{pmma} = \frac{Q_{total}}{m_v}\left(\frac{\bar{W}}{e}\right)_{air}\left.\frac{\left(\frac{\mu_{ab}}{\rho}\right)_{pmma}}{\left(\frac{\mu_{ab}}{\rho}\right)_{air}}\right|_{70keV} \tag{7}$$

The dose to the air. Therefore all that remains is to determine the mass energy-absorption coefficient for Polymethyl Methacrylate at 70 keV. From NIST, this value is $2.416 \times 10^{-2} g/cm^2$.

The table that calculates the mean exposure at the center and the edge can be created from the recorded data in lab, in order to find the overall exposure at the center (which is the current focus).

Table 4: The Table of exposures for the pmma phantom for the center and edge locations.

Trial	1	2	3	4	5	Mean	Variance
Exposure (Center)	792.6 mR	792.7 mR	794.3 mR	793.9 mR	793.9 mR	793.5 mR	0.60 mR^2
Exposure (Edge)	928.0 mR	918.9 mR	948.8 mR	948.4 mR	916.0 mR	932.0 mR	248.7 mR^2

Substituting the mean value at the center and solving gives the dose for the PMMA at the center.

$$D_{pmma}^{center} = \frac{100cm}{40cm}\left(793.5mR \times \frac{1R}{1000mR} \times \frac{2.58\times10^{-4}C/kg}{1R}\right) \times 33.97J/C \times \frac{2.416\times10^{-2}g/cm^2}{2.724\times10^{-2}g/cm^2} = 0.0154Gy$$

This result is the mean, dose and the variance can be found the same way.

$$D_{pmma}^{Var,center} = \frac{100cm}{40cm}\left(\frac{0.60mR^2}{(1000mR)^2} \times \frac{2.58\times10^{-4}C/kg}{1R}\right)\times 33.97J/C\times\left(\frac{2.416\times10^{-2}g/cm^2}{2.724\times10^{-2}g/cm^2}\right)^2 = 1.166\times10^{-8}Gy^2$$

This calculation is the CTDI$_{100}$ dose calculation and is the most commonly used dose estimation in CT exams. Specifically, the dose to air is most commonly referred to the CTDI$_{100}$ measurement.

The CTDI_{100} dose calculation is helpful, though there are some inconsistencies. Therefore a weighted CTDI calculation, which involves finding the dose at both the center of the phantom and at the edge of the phantom. To do this process, the physical meaning of the weighted CTDI needs to be understood. This can be seen when viewed from a mathematical conceptual framework of the average value of dose. By definition, the average dose is given by

$$\bar{D_s} = \frac{E_s}{m_s} = \frac{E_s}{\rho \pi R^2 L} \tag{8}$$

where E_s is the energy absorbed in the volume S. This quantity can be defined by the triple integral

$$E_s = \iiint_S \frac{\partial E(x,y,z)}{\partial m} \rho dx dy dz \tag{9}$$

Changing from cartesian to cylindrical coordinates, the integral becomes:

$$= \rho \int_{-L/2}^{L/2} dx \int_0^{2\pi} d\theta \int_0^R \frac{\partial E}{m}(r) r dr$$

which when simplified becomes

$$E_s = \rho 2\pi L \int_0^R D(r) r dr \tag{10}$$

Note that from this equation, it is assumed that moving along the z-direction has a symmetry. Taking equation (10) and substituting it into equation (8), an expression for the average dose can be found to be:

$$\bar{D_s} = \frac{2}{R^2} \int_0^R D(r) r dr \tag{11}$$

Using the definition of dose as $D(r) = \Psi(r)\frac{\mu_{ab}}{\rho}(r)$, one could arrive at a rigorous definition of the the energy absorbed. However, the average dose being calculated that way is often very difficult, and instead the dose can be calculated as $D(r) \approx D(0)(1 + \alpha r)$, which can be substituted into equation (11).

2.6 Question 6

By substituting $D(r) \approx D(0)(1 + \alpha r)$ into equation (11), one can come to the linear approximation of the average dose, where α is a scaling parameter. The derivation is given below.

$$\bar{D_s} = \frac{2}{R^2} \int_0^R D(r) r dr$$

$$\implies \bar{D_s} = \frac{2}{R^2} \int_0^R D(0)(1 + \alpha r) r dr$$

$$\implies \bar{D_s} = \frac{2D(0)}{R^2} \int_0^R (r + \alpha r^2) dr$$

$$\implies \frac{2D(0)}{R^2} \left[\frac{1}{2}r^2 + \frac{1}{3}\alpha r^3\right]_0^R$$

$$\implies \frac{2D(0)}{R^2} \left(\frac{1}{2}R^2 + \frac{1}{3}\alpha R^3\right)$$

$$\implies 2D(0) \left(\frac{1}{2} + \frac{1}{3}\alpha R\right)$$

$$\implies D(0) \left(1 + \frac{2}{3}\alpha R\right)$$

$$\implies D(0) \left(\frac{1}{3} + \frac{2}{3} + \frac{2}{3}\alpha R\right)$$

$$\implies D(0)\left(\frac{1}{3}+\frac{2}{3}(1+\alpha R)\right)$$

$$\implies \bar{D_s}=\frac{1}{3}D(0)+\frac{2}{3}D(R) \tag{12}$$

In order to use this equation, the dose at the edge must also be determined. This process, however, is something that has already done before, and the same process as the one used to find the dose at the center of the phantom can be used. Using the value from Table 3, the dose at the edge can be found to be:

$$D_{edge}^{center}=\frac{100cm}{40cm}\left(932.0mR\times\frac{1R}{1000mR}\times\frac{2.58\times10^{-4}C/kg}{1R}\right)\times 33.97J/C\times\frac{2.416\times10^{-2}g/cm^2}{2.724\times10^{-2}g/cm^2}=0.0181Gy$$

Substituting these values into the equation for the CTDI$_{weighted}$ expression, equation (12), a result can finally be obtained.

$$CTDI_{Weighted}=\frac{1}{3}(0.0154Gy)+\frac{2}{3}(0.0181Gy)=0.0172Gy \tag{13}$$

And that's the calculation of the weighted CTDI.